This teaching unit is designed to help students in grades 4-8 explore the concepts of algebra in the context of exploring the universe. The units in this series have been developed to enhance and enrich mathematics, science, and technology education and to accommodate different teaching and learning styles. Each unit consists of a storyline presenting the context for the problems to be solved, lists of the mathematics and science concepts addressed, background notes for the teacher, a list of teacher resources, and an activity complete with blackline masters. Also included are suggestions for extensions to the problems and their relationship to national mathematics standards. The storyline for this unit is students learning how algebra is used to explore the universe. (MM)
Algebra:

Mirror, Mirror on the Universe

- **Story Line:** Students will learn how algebra is used to explore the universe.
- **Math Concepts:** Estimations, Expressions, Solving Equations, Word Problems.
- **Science Concepts:** Matter, Temperature, Chemical and Physical Changes, Light
- **NASA Research:** Hubble Space Telescope, Next Generation Space Telescope

**Educator's Guide**

<table>
<thead>
<tr>
<th>Teachers &amp; Students</th>
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</table>

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PROGRAM SUMMARY

OBJECTIVE

In *Algebra: Mirror, Mirror on the Universe*, students will discover how algebra and telescopes are used in space exploration and why optics, which is the study of light, is important in astronomy. Students will learn about the Hubble Space Telescope, Hubble Deep Field, and how NASA engineers use algebra to determine the effects of contamination on Hubble's optics. They will also see how NASA engineers are developing the Next Generation Space Telescope (NGST) with the challenge of making bigger instruments with ultralight-weight materials able to protect NGST's optics from the heat of the Sun. By participating in the classroom activity, students will gain insight into the concept of the expanding universe.

CLASSROOM ACTIVITY

Students will participate in a classroom activity to apply what they have learned about the expansion of space and linear relationships. This activity requires the students to create a model that demonstrates the relationship between the distance of an object and its recession speed. Students will measure, collect data, graph, and interpret the graphs to understand the effects of expansion. They will compare the results of their graphs with the graph of the Hubble's results to aid in their understanding of the concept of space expansion. For more information on *Algebra: Mirror, Mirror on the Universe*, visit the NASA CONNECT web site: edu.larc.nasa.gov/connect/mirror.html

WEB-BASED COMPONENT

Do you want to take your class on a field trip to the edge of the observable universe? Visit Amazing Space: Hubble Deep Field Academy. This online activity is located in Norbert's Lab at edu.larc.nasa.gov/connect/mirror/norbert/lab.html

Amazing Space: Hubble Deep Field Academy is the online activity that provides an opportunity for students to join Professor Wipic and the cadets of the Hubble Academy as they count, classify, and analyze objects from the Hubble Deep Field, almost 12 billion light-years away. In this lesson, students will begin to comprehend the vastness of the universe by examining the Hubble Deep Field image and simulating the process astronomers use to count, classify, and identify the objects in the image. Students are challenged to complete four activities and achieve the rank of Universal Graduate of the Hubble Deep Field Academy.
CAREER CORNER

Access to information is critical to making career decisions. Career Corner, located in Norbert's Lab at edu.larc.nasa.gov/connect/mirror/norbert/lab.html, is a web-based component that highlights the professionals who appear in the program Algebra: Mirror, Mirror on the Universe. This web site includes pictures of the professionals; summarizes their duties and responsibilities; and includes details about the person, event, or situation that greatly influenced their career choice.

TEACHER BACKGROUND

HUBBLE’S MEASUREMENTS

By the early 1920s, Vesto Slipher had been able to determine that distant objects were moving rapidly, and later he found that most of them were moving away from us at very high speeds. Edwin Hubble discovered that many of these distant objects were vast collections of stars, which we now call galaxies. Galaxies can contain billions or trillions of stars and be hundreds of thousands of light years across. It is estimated that there are about a trillion galaxies in the observable universe. A light-year is the distance that light travels in one year. Our own Milky Way is such a galaxy of stars. It contains around 100 billion stars and is about 120 light-years across. Hubble was also able to determine the distance of many of these objects and with his collaborators, such as Milton Humason, found that the further away an object is from us, the faster it appears to be moving away from us (receding). The relationship can be expressed as an algebraic equation: V = HD, where V is the recession velocity of the object from Earth, D is the distance of the object, and H is Hubble’s constant.

THE EXPANDING UNIVERSE

The model that scientists use to explain the apparent motion of the galaxies is that the space, in which all objects find themselves, is expanding. You can imagine raisins in a rising loaf of bread in which the bread is space and the raisins are far away objects like galaxies. We can’t visualize some of the details as easily in this three dimensional space, but we can imagine an analogous process in two-dimensional space. Please note, this visualization in two dimensions is an analogy. It is not the real thing, but scientists often have to use analogies to understand tough concepts.

Two types of two-dimensional space are flat space (a tabletop) and curved space (the surface of a balloon). Creatures in two-dimensional space have no idea of the third dimension. For example, they can’t picture up and down if they are confined to a tabletop because they can’t leave the surface of the table. Locations outside the surface of the tabletop don’t exist for them. There is nothing besides the surface; space is only the surface of the
tabletop in this analogy. To visualize the expansion of space, imagine two points on the surface of the balloon (which we will say represents space). You can see the points moving apart as the balloon (or space) expands. This expansion is the source of the recession of distant objects and is caused by the expansion of space, not by movement through space. If you look carefully at the points on the surface of the balloon, you will see that the points that are farther from you are receding faster because they separate more when the balloon is expanded.

This analogy also helps us understand the puzzling observation that most distant objects seem to be moving away from us. How could we just accidentally be at the center of the universe? Several times in history, going back to the Greeks, we mistakenly believed we were at the center of the universe. Think about the surface of the balloon. There is no center — no point is anymore important than any other. No matter where we are, it will look as if everything is moving away from us as the balloon (or space) expands. We aren’t accidentally at the center because there is no single center, and we will see everything recede from us no matter where we are. By the way, the objects that are moving towards us are moving through space — there are always some renegades.

Around 1929, the Belgian priest, Georges Lemaître, proposed the idea of an explosive beginning for the universe. In 1948, the Russian physicist, George Gamow, (pronounced gam off) pursued the idea further and coined the phrase “big bang.” The big bang theory is one theory among many that scientists explore. The big bang theory states that everything in the universe, including space, came from an unimaginably hot, dense, singular point.

**NATIONAL MATH STANDARDS**

- Number and Operation
- Patterns, Functions, and Algebra
- Geometry and Spatial Sense
- Measurement
- Data Analysis, Statistics, and Probability
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation
NATIONAL SCIENCE STANDARDS

- Unifying Concepts and Processes
- Science as Inquiry
- Earth and Space Science
- History and Nature of Science

NATIONAL TECHNOLOGY STANDARDS

- Basic Operations and Concepts
- Technology Research Tools
- Technology Problem-Solving and Decision-Making Tools

INSTRUCTIONAL OBJECTIVES

Students will
- be challenged to think about the meaning of space.
- learn about Hubble's work relating to the expansion of space.
- interpret a graph.
- gain understanding of the algebraic concept of linear relationship.
- work effectively in small groups to take accurate measurements.
- plot and analyze the results of their measurements.
- use the idea of model to understand the data.
- use analogy to understand concepts.

TEACHER RESOURCES

Books
Web Sites

NASA SpaceLink: Hubble Space Telescope
Telescope/

Exploratorium Observatory - Project Serendip; The Search for Life
http://www.exploratorium.edu/observatory/index.html

The NASA CERES Project - The Expanding Universe activity
http://btc.montana.edu/ceres/html/uni1.html

Space Kids - Participate in making astronomical observations. Take an interactive tour of the solar system.
http://spacekids.hq.nasa.gov/

Space Telescope Science Institute - Current Hubble Statistics, images, and activities
http://www.stsci.edu

Space Telescope Science Institute - Hubble: images and animations
http://www.oposite.stsci.edu/pubinfo/Anim.html

Discovery Channel - Hubble Lesson Plans

PBS Teachersource™ Mathline - Activity on calculating a light year
http://www.pbs.org/teachersource/mathline/concepts/space2/activity1.shtml
algebra - a tool used by mathematicians to describe patterns and relationships with symbols
analogy - a similar or like situation often used to explain a difficult concept
center - the middle; the center of a circle is the same distance from all points on the circumference
dimensions - different directions in space such as forward, sideways, and up for our three-dimensional world
distance - how far it is from one point to another as measured by a ruler
epoch - a period of time considered in terms of noteworthy and characteristic events
expansion rate - how rapidly something expands; the amount expanded divided by the time it takes
galaxy - a collection of stars attracted to each other by gravity; often billions of stars
galaxy - an attractive force between stuff the physicists call mass
light year - the distance light travels in one year
linear relationship - when one quantity changes in the same proportion as another; for example, if water runs from a faucet at a steady rate, the amount of water collected in a cup will double if the time doubles and the amount of water collected is proportional to or linearly related to the time
measurement unit - a standard quantity agreed upon by many people, such as the second, which is used to measure time
Milky Way - our galaxy
millimeter - a measurement unit of distance about the thickness of your fingernail
model - a simple or complex plan dreamed up to explain observations
patterns - an organized relationship between objects such as the cells of a honeycomb
recession - moving away
relationships - connections between quantities
solar system - the Sun and all the planets, comets, asteroids, and space objects bound to the Sun by the force of gravity
space - the expanse in which the solar system, stars, and galaxies exist
speed - distance moved per unit of time
star - a big ball of burning gas held together by gravity
Sun - the closest star to Earth
time - actually not definable, the connection between past, present and future measured by a clock
universe - all existing things, including the Earth, the heavens, the galaxies, and all therein
velocity - speed in a given direction
THE ACTIVITY: HUBBLE AND THE EXPANDING UNIVERSE

BEFORE THE ACTIVITY

Encourage the students to research the Hubble constant, the big bang theory, and the expansion of space by using the library, the Internet, and the resources listed in this guide. Challenge the students to think about and discuss (or write a response to) each of the following questions:

- What is space?
- How big is the observable universe?
- When we observe far away galaxies, do we see them as they are now?
- What did Hubble measure?
- What is the Hubble constant?
- What is the big bang theory?
- What do we mean when we say that space is expanding?

THE ACTIVITY

In this activity, students will examine the algebraic relationship between recession speed (expansion rate) of distant objects (e.g., galaxies) and the object's distance from us. Students will use an analogy between our actual space and two-dimensional space to take measurements showing that the relationship is approximately linear. The results of the activity show that the expansion of space is a plausible explanation for the results of Hubble's measurements.

What is this stuff that is to our right, our left, behind us, in front of us, above us, and even under us? Space! Space is what we exist in, live in, and move around in. What is distance? The amount of space between two objects measured in some standard set of units is called space. What is expansion? When something expands, it grows larger. If space expanded, what would happen to two objects in that space (imagine raisins in a loaf of rising bread)? The two raisins would move farther apart.

In the 1920s, Vesto Slipher had been able to determine that distant objects were moving rapidly, and most of them were moving away from us at very high speeds. Edwin Hubble was able to determine the distance from Earth of many of these objects, and with his collaborators, like Milton Humason, found that the farther away an object is from us, the faster it is moving away from us (receding).
MATERIALS

balloons - round, approximately 11 inches
ruler or straight edge
measuring tape or string and meter stick
binder clips for closing off the neck of the balloon
marking pen - fresh, sharp, permanent
graph paper
Universe Data Sheet, page 14
paper for calculations
pencil
calculator

1. The Hubble Constant and Algebra

Create a transparency of the Hubble Data Chart on page 15, or distribute paper copies of the chart to students.

Discuss the parts of the graph.
1. The horizontal axis represents distance.
2. The vertical axis represents recession speed.
3. What pattern do you see? The patterns lie in a straight line.

Have the students interpret the data from the chart.
1. What is the velocity (v, recession speed) of the object when the distance (D) is the small value of D?
2. What is the velocity of the object when the distance is the large value of D?
3. What statement can you make about the relationship between distance and velocity?
4. Draw the best straight line you can through the data points.
5. What is the predicted approximate value of the velocity when D is 2000?

The relationship between velocity and distance in the chart can be expressed as an algebraic equation:

\[ V = H \cdot D \]

is a mathematical formula similar to \[ A = l \times w \]

\( V \) = the recession velocity of the object (how fast the object is moving away)

\( D \) = the distance of the object from a specific point

\( H \) = Hubble's constant, a value that has yet to be determined exactly.

Mathematicians use algebra to describe patterns like Hubble's data. \( V = H \cdot D \) is the algebraic description of the Hubble pattern. Scientists use models to explain observations. The model used to explain Hubble's data is that space is expanding. Imagine raisins in a rising loaf of bread: the bread is space and the raisins are far away objects like galaxies. We can't do an experiment in Earth space with the loaf of rising bread because it is too difficult, but we can do an analogous experiment in two-dimensional space with results that will behave the same as in Earth space. Scientists often use analogies to understand tough concepts.

Let's explore two different two-dimensional spaces - flat space (the tabletop) and curved space (the surface of a balloon). Creatures in two-dimensional space have no idea of the third dimension; for example, up and down if we talk about the tabletop. They can't leave the surface of the table. The surface outside doesn't exist for them. What is inside the balloon? Nothing, space is only the surface of the balloon in this analogy.
2. Demonstration in Flat Space  (Teacher demonstration)

Prepare the following:

- Make one copy of the Dots-Galaxies Worksheet on page 16.
- From the original copy, reproduce three copies: one at 75% smaller than original, one at 50% smaller than original, and one at 150% larger than original.
- Make a transparency of each copy.
- Begin by placing the smallest reproduction (50%) on an overhead projector. Explain to students that the dots represent a collection of distant objects like galaxies.
- Pick any point (1-5) as home and place the 75% reproduction transparency directly on top, making sure the home dot from the first transparency is aligned beneath the home dot of the second transparency. Note: Other dots will not line up now. For example, if you picked 1 as the home dot, make sure you continue to line up the 1 on each successive transparency.
- Ask students, “What do you observe from the first transparency to the second?” Students should observe that the dots are moving away from the home dot.
- Continue by placing the original transparency on top of the 75% reproduction, once again aligning the home dots. Discuss observations.
- Finally, place the largest reproduction (150%) on top, making sure the home dot is aligned with the rest.

Point out that each successive transparency represents an expansion of flat two-dimensional space. It should be obvious that everything appears to fly away from the home dot. Repeat this demonstration using another home dot.

What did you observe? You should see that no matter which dot you choose as your home dot, all the other dots appear to fly away from it.

What does this demonstration show us about the center of the universe? This demonstration should illustrate that there is no center to the universe!

3. Activity in Curved Space  (Student activity)

Divide the class into small research teams of at most five students. Make sure each member is assigned at least one of the following tasks:

- a. expanding the balloon
- b. marking the balloon
- c. measuring
- d. checking measurements
- e. recording
- f. calculating
- g. plotting
- • Distribute the materials (keep this list handy).
- Expand the balloon to about 400 mm circumference (grapefruit size). Roll the neck of the balloon, making three turns, toward the expanded portion. Secure with a binder clip to keep air from escaping. See Figure 1. Label the surface of the balloon “SPACE.” Mark a sharp point near the balloon’s equator (think of the neck of the balloon as the North Pole). Label this first point as home.
- Starting from home, measure 10 mm intervals along the balloon’s equator and mark 5 sharp points (galaxies look like points to us). Don’t compress or dent the balloon while making the marks.
- Label each point starting with the number 1.
- Measure again the distance to point number 1 from home to be sure no air has escaped. Record on the Universe Datasheet (page 14) the distance from home to each point.
- Expand the balloon from the grapefruit size of 400 mm to about the size of your head, 600 mm.
- Roll the neck of the balloon and secure as previously indicated.
- Measure the new distance from home to each point (galaxy) and record the results on the Universe Datasheet (page 14). Don’t compress or dent the balloon while making the measurements. Have someone check the measurements. Wise carpenters always say “measure twice, cut once.”
- Check for leaks by measuring again the distance of each point.
- Calculate the distance the point moved by subtracting the first recorded distance from home, from the second recorded distance. Have someone check the calculations. Record the results in the Distance traveled column on the Universe Datasheet (page 14).

Second distance from home - Starting distance from home = Distance traveled

- Divide the distance each point traveled by the time it took (1 epoch) to get the expansion rate. This answer is the rate of expansion of your balloon. Record the result for each point on the Universe Datasheet (page 14).

4. Plot the Results
Prepare the graph:
1. Draw a vertical and a horizontal axis on graph paper with 5-mm spacings between gridlines (see example).
2. Label the horizontal axis as Distance.
3. Label the vertical axis as Expansion Rate. (Expansion rate is directly related to the velocity of recession that astronomers measure.)
Plot the points:
1. Using the data from the Universe Datasheet (page 14), plot the points (starting distance from home, expansion rate.)
2. Team members should verify that the points are plotted correctly on the graph.

5. Analyze the results
What do we have? An approximate straight line! This result agrees with Hubble’s observations; therefore, this two-dimensional analogy tells us that this model of space expanding is a good one.
• Does the pattern of your graph resemble the pattern of the graph of Hubble's observations we saw in the first part of the activity? What do mathematicians call this pattern? (Linear)

• What is the relationship between the points on the balloon and objects in space?

• Your results weren't a perfect straight line. What reasons can you give for this?

• What does it mean when we say the universe is expanding?

• Does the concept of a single “center” make sense for the universe? Explain.

• Can you estimate the distance of a sixth point on your balloon? Explain your method.

EXTENSION ACTIVITIES

Using the concept described in part 2 of the Classroom Activity, create paper copies of the transparencies.

• Distribute a set of copies to each student (or group of students).

• Choose a home from the dots on the paper that has the smallest grouping. Each student may choose a different home.

• Layer the pages in the same order as demonstrated earlier. Identify the “path” each point has taken. What observation can you make about the apparent distances traveled of each point from its original position?

• Put all papers to the side except the paper with the smallest grouping (50%).

• Measure the distance from home to each of the other five dots. Record the distances.

• Use the paper with the largest grouping and measure the same pairs (home to each point). Record the distance.

• Subtract the first distance recorded from the last distance recorded for each point.

• What did you find? Is the distance greater for points farther away? How does this result compare to the balloon observation?

Using the graph provided in part 1 of the Classroom Activity, determine the following:

1. The approximate value of the Hubble Constant, given the information in the graph.

According to the information provided in the Teacher Resource Guide, the Hubble Constant (H) is expressed in the relationship between recession velocity of the object and the distance of the object, \( V = HD \).

Use the formula for slope to calculate an approximate value for H.

• Choose any two points on the graph. Record the coordinates.

• Determine the slope between the two points using \( m = (Y' - Y)/(X' - X) \)

• Record m as your value for the Hubble Constant. (Hubble Constant is approx: 15)

How does your value compare to Hubble estimates found by other students? Why are the values different when everyone is using the same graph?
2. Use the Hubble Constant found in problem 1 above to write the equation of a straight line representing the graph.
   - Assume the line has a y-intercept of 0; write the equation. \( V = 15D \)

3. Using the equation found in problem 2 above, predict the approximate recession velocity \( V \) of an object that has a distance of 2200 light years.
   - Substitute the value of 2200 for the variable \( D \). \( V = 15 \times 2200 \).
   - Simplify. \( V \approx 33,000 \).

Does your answer make sense? Why or why not?
What is optics and how is algebra used in optics?

How do engineers take care of the Hubble Space Telescope while it's in space?

Compare and contrast the Hubble Space Telescope to the Next Generation Space Telescope.
**UNIVERSE DATA SHEET**

<table>
<thead>
<tr>
<th>Points</th>
<th>Starting distance from home (*space units)</th>
<th>Second distance from home (space units)</th>
<th>Distance traveled (space units)</th>
<th>Expansion rate (space units/epoch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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*Define the distance units: one millimeter = one space unit. After all, we are working on a two-dimensional curved surface of a balloon, which is only an analogy to the real case.

Note: It is difficult in this activity to time the expansion of the balloon in terms of minutes or seconds (some people can inflate a balloon faster than others). Therefore, in this activity, time will be described in terms of a unit called an epoch (see vocabulary list, page 6). One epoch equals one expansion.

1. Find the distance traveled.
   The distance traveled is the difference between the second distance from home and the starting distance from home.

   \[ \text{second distance} - \text{starting distance} = \text{distance traveled} \]

2. Determine the expansion rate.
   The expansion rate is the distance traveled divided by time (1 epoch).

   \[ \frac{\text{distance traveled}}{\text{time}} = \text{expansion rate} \]

The resulting value is the expansion rate in space units per epoch.
HUBBLE DATA CHART

Hubble Constant Data

Recession Velocity (\(v\)) in Km/s

Distance (D) in light years

[Diagram showing points on a grid with axes labeled]
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